10.1 THE CIRCLE

Objectives

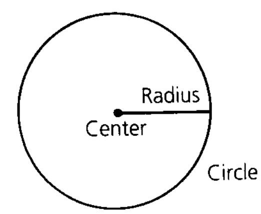
After studying this section, you will be able to
- Identify the characteristics of circles
- Recognize chords and diameters of circles
- Recognize special relationships between radii and chords

Part One: Introduction

Basic Properties and Definitions

The following definitions will help you extend and organize what you already know about circles.

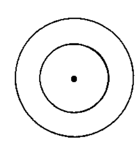
Definition A ***circle*** is the set of all points in a plane that are a given distance from a given point in the plane. The given point is the ***center*** of the circle, and the given distance is the ***radius***. A segment that joins the center to a point on the circle is also called a radius. (The plural of *radius* is *radii*.)



The definitions of *circle* and *radius* can be used to prove a theorem you saw in Chapter 3: *All radii of a circle are congruent* (Theorem 19).

Although all circles have the same shape, their sizes are determined by the measures of their radii.

Definition Two or more coplanar circles with the same center are called ***concentric*** circles.

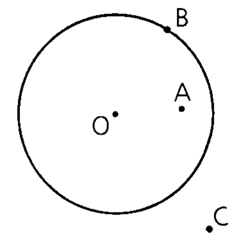


Definition Two circles are congruent if they have congruent radii.

Definition A point is inside (in the ***interior*** of) a circle if its distance from the center is less than the radius.

Points O and A are in the interior of ⊙O.

Definition A point is outside (in the ***exterior*** of) a circle if its distance from the center is greater than the radius.



Point C is in the exterior of ⊙O.

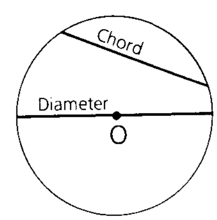
Definition A point is on a circle if its distance from the center is equal to the radius.

Point B is on ⊙O.

Chords and Diameters

Points on a circle can be connected by segments called ***chords.***

Definition A ***chord*** of a circle is a segment joining any two points on the circle.



What is the longest chord of a circle? Is there a shortest chord?

Definition A ***diameter*** of a circle is a chord that passes through the center of the circle.

The ideas of circumference and area of a circle are important in geometry. We now review two formulas presented in Chapter 3.

Circumference and Area of a Circle

The area of a circle can be found with the formula

$$A = \pi r^2$$

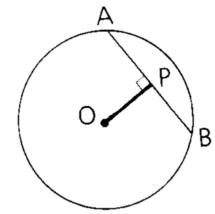
and the circumference (perimeter) of a circle can be found with the formula

$$C = \pi d$$

where r is the circle's radius, d is its diameter, and $\pi \approx 3.14$.

Radius-Chord Relationships

OP is the distance from O to chord $\overline{AB}$.

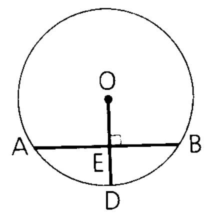


Definition The distance from the center of a circle to a chord is the measure of the perpendicular segment from the center to the chord.

The following three theorems are useful in establishing special relationships between radii and chords.

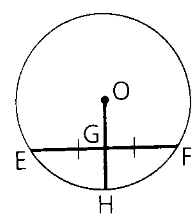
Theorem 74 ***If a radius is perpendicular to a chord, then it bisects the chord.***

Given: $\odot O$,
$\overline{OD} \perp \overline{AB}$
Prove: $\overline{OD}$ bisects $\overline{AB}$.



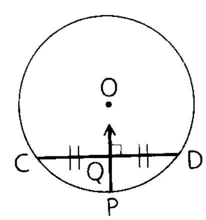
Theorem 75 ***If a radius of a circle bisects a chord that is not a diameter, then it is perpendicular to that chord.***

Given: $\odot O$;
$\overline{OH}$ bisects $\overline{EF}$.
Prove: $\overline{OH} \perp \overline{EF}$



Theorem 76 ***The perpendicular bisector of a chord passes through the center of the circle.***

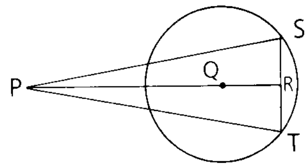
Given: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{CD}$.
Prove: $\overleftrightarrow{PQ}$ passes through O.



Part Two: Sample Problems

Problem 1

Given: ⊙Q,
$\overline{PR} \perp \overline{ST}$
Prove: $\overline{PS} \cong \overline{PT}$

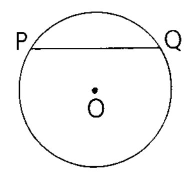


Proof

1 ⊙Q, $\overline{PR} \perp \overline{ST}$	1 Given
2 $\overline{PR}$ bisects $\overline{ST}$.	2 If a radius is ⊥ to a chord, it bisects the chord. ($\overline{QR}$ is part of a radius.)
3 $\overline{PR}$ ⊥ bis. $\overline{ST}$	3 Combination of steps 1 and 2
4 $\overline{PS} \cong \overline{PT}$	4 If a point is on the ⊥ bis. of a segment, then it is equidistant from the endpoints.

Problem 2

The radius of circle O is 13 mm. The length of chord $\overline{PQ}$ is 10 mm. Find the distance from chord $\overline{PQ}$ to the center, O.

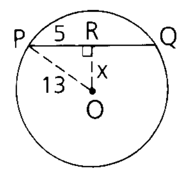


Solution

Draw $\overline{OR}$ perpendicular to $\overline{PQ}$.
Draw radius $\overline{OP}$ to complete a right △.
Since a radius perpendicular to a chord bisects the chord,

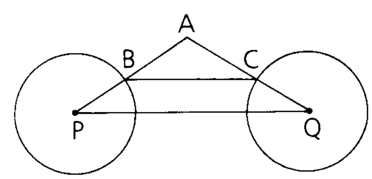
$$PR = \tfrac{1}{2}(PQ) = \tfrac{1}{2}(10) = 5$$

By the Pythagorean Theorem,
$x^2 + 5^2 = 13^2$, so OR = 12.



Problem 3

Given: △ABC *is isosceles* ($\overline{AB} \cong \overline{AC}$).
ⓢ P *and* Q,
$\overline{BC} \parallel \overline{PQ}$
Prove: ⊙P ≅ ⊙Q



Proof

1 △ABC is isosceles ($\overline{AB} \cong \overline{AC}$).	1 Given
2 ⓢ P and Q, $\overline{BC} \parallel \overline{PQ}$	2 Given
3 ∠ABC ≅ ∠P, ∠ACB ≅ ∠Q	3 ∥ lines ⇒ corr. ∠s ≅
4 ∠ABC ≅ ∠ACB	4 If △, then △.
5 ∠P ≅ ∠Q	5 Transitive Property
6 $\overline{AP} \cong \overline{AQ}$	6 If △, then △.
7 $\overline{PB} \cong \overline{CQ}$	7 Subtraction (1 from 6)
8 ⊙P ≅ ⊙Q	8 ⓢ with ≅ radii are ≅.

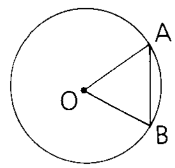
Part Three: Problem Sets

Problem Set A

1 Given: $\odot O$, chord $\overline{AB}$

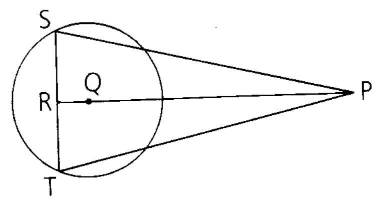
Prove: **a** $\triangle AOB$ is isosceles.

b $\angle A \cong \angle B$



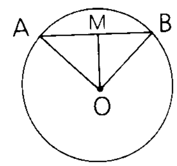
2 Given: $\odot Q$, $\overline{PR} \perp \overline{ST}$

Prove: $\angle S \cong \angle T$



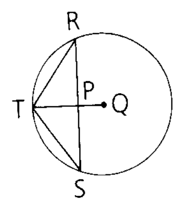
3 Given: $\odot O$; $\overline{OM}$ is a median.

Conclusion: $\overline{OM}$ is an altitude.

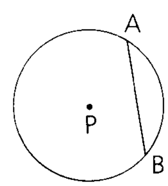


4 Given: $\odot Q$, $\overline{QT} \perp \overline{RS}$

Prove: $\overrightarrow{TQ}$ bisects $\angle RTS$.



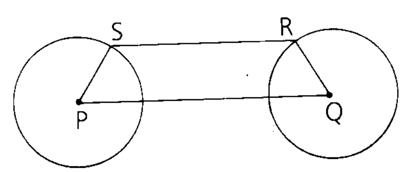
5 Chord $\overline{AB}$ measures 12 mm and the radius of $\odot P$ is 10 mm. Find the distance from $\overline{AB}$ to P.



6 Find the length of a chord that is 15 cm from the center of a circle with a radius of 17 cm.

7 Given: PQRS is an isosceles trapezoid, with $\overleftrightarrow{SR} \parallel \overleftrightarrow{PQ}$.

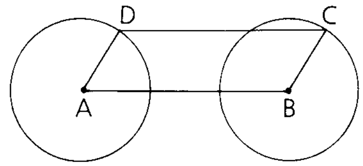
Conclusion: $\odot P \cong \odot Q$



8 Find, to the nearest tenth, the circumference and the area of a circle whose diameter is 7.8 cm.

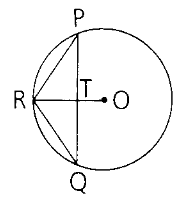
Problem Set A, *continued*

9 Given: $\odot A \cong \odot B$,
$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$
Prove: ABCD is a $\square$.



Problem Set B

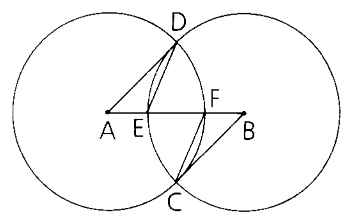
10 Given: $\odot O$;
$\overleftrightarrow{OR}$ bisects $\overline{PQ}$.
Prove: $\overrightarrow{RO}$ bisects $\angle PRQ$.



11 Find the distance from the center of a circle to a chord 30 m long if the diameter of the circle is 34 m.

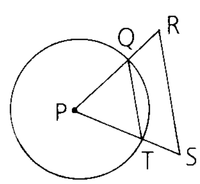
12 Find the radius of a circle if a 24-cm chord is 9 cm from the center.

13 Given: Ⓢ A and B intersect as shown.
$\overline{DE} \parallel \overline{FC}$, $\angle ADE \cong \angle FCB$,
$\overline{DE} \cong \overline{FC}$
Prove: $\odot A \cong \odot B$



14 Two circles intersect and have a common chord 24 cm long. The centers of the circles are 21 cm apart. The radius of one circle is 13 cm. Find the radius of the other circle.

15 Given: $\odot P$,
$\overleftrightarrow{QT} \parallel \overleftrightarrow{RS}$
Conclusion: $\overline{QR} \cong \overline{TS}$



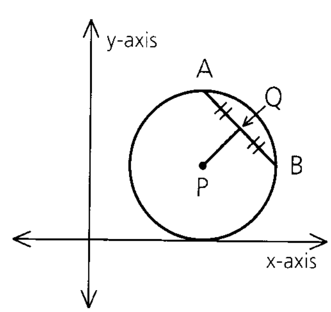
16 $\overline{PQ}$ is a diameter of $\odot O$. P = (−3, 17) and Q = (5, 2). Find the center and the radius of $\odot O$.

17 $\odot P$ just touches (is tangent to) the x-axis. P = (15, 13) and Q = (19, 16).

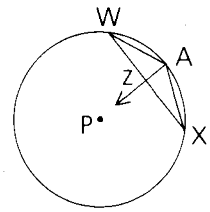
a Find the radius of $\odot P$.

b Find PQ.

c Find the length of $\overline{AB}$.

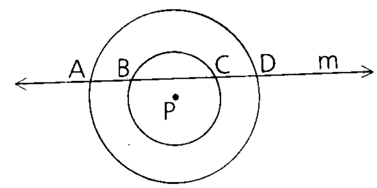


18 Given: $\odot P$;
Z is the midpt. of $\overline{WX}$.
$\triangle WAX$ is isosceles, with base $\overline{WX}$.
Prove: $\overrightarrow{AZ}$ passes through P.

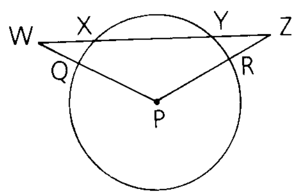


Problem Set C

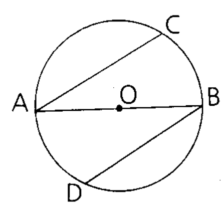
19 Given: Two concentric circles with center P. Line m intersects the circles at A, B, C, and D.
Conclusion: $\overline{AB} \cong \overline{CD}$



20 Given: $\odot P$, $\overline{WX} \cong \overline{YZ}$
Prove: $\overline{WQ} \cong \overline{ZR}$

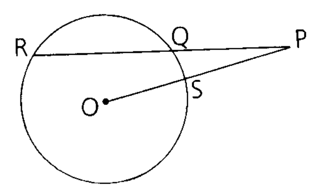


21 Given: $\overline{AB}$ is a diameter of $\odot O$.
$\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$
Conclusion: $\overline{AC} \cong \overline{BD}$



22 Find the radius of a circle in which a 48-cm chord is 8 cm closer to the center than a 40-cm chord.

23 In circle O, PQ = 4, RQ = 10, and PO = 15. Find PS (the distance from P to $\odot O$).



24 An isosceles triangle with each leg measuring 13 is inscribed in a circle. If the altitude to the base of the triangle is 5, find the radius of the circle.

25 Two circles intersect and have a common chord. The radii of the circles are 13 and 15. The distance between their centers is 14. Find the length of their common chord.

10.2 CONGRUENT CHORDS

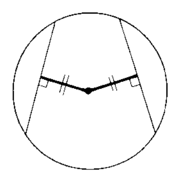
Objective

After studying this section, you will be able to

- Apply the relationship between congruent chords of a circle

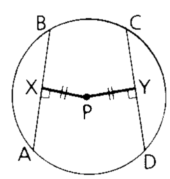
Part One: Introduction

If two chords are the same distance from the center of a circle, what can we conclude?



Theorem 77 ***If two chords of a circle are equidistant from the center, then they are congruent.***

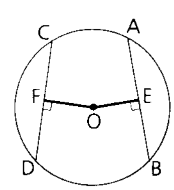
Given: $\odot P$, $\overline{PX} \perp \overline{AB}$, $\overline{PY} \perp \overline{CD}$, $\overline{PX} \cong \overline{PY}$
Prove: $\overline{AB} \cong \overline{CD}$



The proof of Theorem 77 is left for you to do. (Use four congruent triangles.) The converse of Theorem 77 can also be proved.

Theorem 78 ***If two chords of a circle are congruent, then they are equidistant from the center of the circle.***

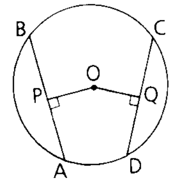
Given: $\odot O$, $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$
Prove: $\overline{OE} \cong \overline{OF}$



Part Two: Sample Problems

Problem 1 Given: $\odot O$, $\overline{AB} \cong \overline{CD}$,
$OP = 12x - 5$, $OQ = 4x + 19$

Find: OP



Solution Since $\overline{AB} \cong \overline{CD}$, $OP = OQ$.

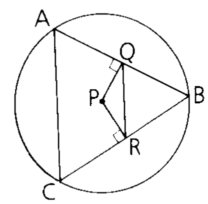
$12x - 5 = 4x + 19$

$x = 3$

Thus, $OP = 12(3) - 5 = 31$.

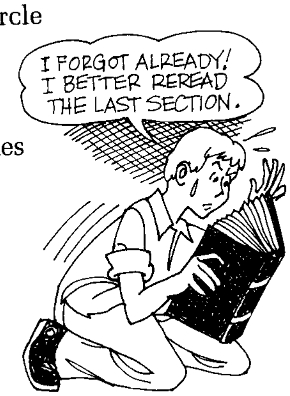
Problem 2 Given: $\triangle ABC$ is isosceles, with base $\overline{AC}$.
$\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$

Prove: $\triangle PQR$ is isosceles.



Proof

1 $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$	1 Given
2 $\triangle ABC$ is isosceles, with base $\overline{AC}$.	2 Given
3 $\overline{AB} \cong \overline{BC}$	3 An isosceles $\triangle$ has two $\cong$ sides.
4 $\overline{PQ} \cong \overline{PR}$	4 If two chords of a circle are $\cong$, then they are equidistant from the center.
5 $\triangle PQR$ is isosceles.	5 A $\triangle$ with two $\cong$ sides is isosceles.



Why do you think it was necessary to be given $\overline{PQ} \perp \overline{AB}$ and $\overline{PR} \perp \overline{CB}$, even though they did not seem to play an active role in the proof?

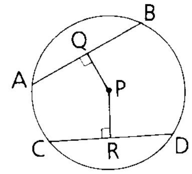
Part Three: Problem Sets

Problem Set A

1 In a circle, chord $\overline{AB}$ is 325 cm long and chord $\overline{CD}$ is $3\frac{1}{4}$ m long. Which is closer to the center?

2 Given: $\odot P$, $\overline{PQ} \cong \overline{PR}$,
$AB = 6x + 14$,
$CD = 4 - 4x$

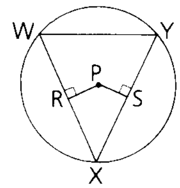
Find: AB



Problem Set A, *continued*

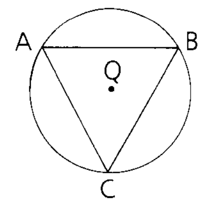
3 Given: ⊙P, $\overline{PR} \perp \overline{WX}$, $\overline{PS} \perp \overline{XY}$, $\overline{PR} \cong \overline{PS}$

Conclusion: $\angle W \cong \angle Y$



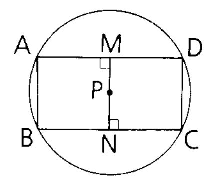
4 Given: Equilateral △ ABC is inscribed in ⊙Q.

Conclusion: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are equidistant from the center.



5 Given: ⊙P; P is the midpoint of $\overline{MN}$. $\overline{MN} \perp \overline{AD}$, $\overline{MN} \perp \overline{BC}$

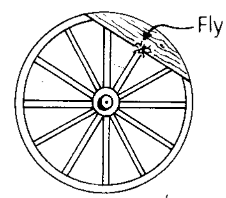
Conclusion: ABCD is a ▱.



6 A fly is sitting at the midpoint of a wooden chord of a circular wheel. The wheel has a radius of 10 cm, and the chord has a length of 12 cm.

a How far from the hub (center) is the fly?

b The wheel is spun. What is the path of the fly?

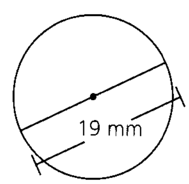


Problem Set B

7 To the nearest hundredth, find

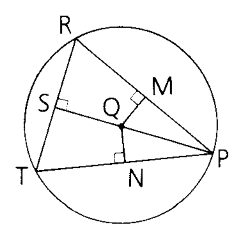
a The area of the circle

b The circumference of the circle



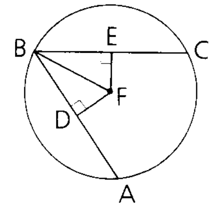
8 Given: ⊙Q, $\overline{PS} \perp \overline{RT}$, $\overline{MQ} \perp \overline{RP}$, $\overline{NQ} \perp \overline{PT}$

Conclusion: $\overline{MQ} \cong \overline{QN}$



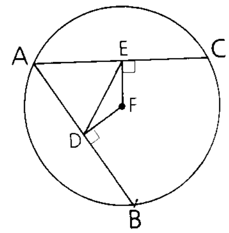
9 Given: ⊙F, $\overline{FE} \perp \overline{BC}$, $\overline{FD} \perp \overline{AB}$; $\overrightarrow{BF}$ bisects $\angle ABC$.

Prove: $\overline{BC} \cong \overline{BA}$



10 Given: $\odot F$, $\overline{AB} \cong \overline{AC}$, $\overline{DF} \perp \overline{AB}$, $\overline{EF} \perp \overline{AC}$

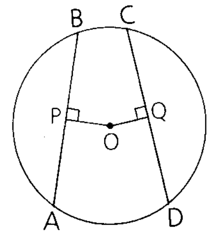
Prove: $\triangle ADE$ is isosceles.



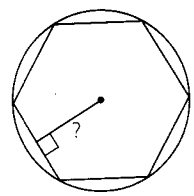
11 In circle O, $PB = 3x - 17$, $CD = 15 - x$, and $OQ = OP = 3$.

a Find AB.

b Find the radius of $\odot O$.



12 A regular hexagon with a perimeter of 24 is inscribed in a circle. How far from the center is each side?

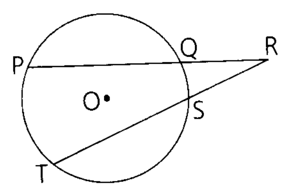


13 A 16-by-12 rectangle is inscribed in a circle. Find the radius of the circle.

Problem Set C

14 Given: $\odot O$, $\overline{PQ} \cong \overline{TS}$

Prove: $\overline{RQ} \cong \overline{RS}$



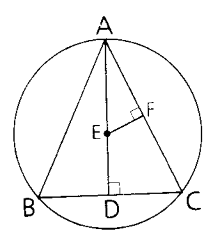
15 Given: $\triangle ABC$ is isosceles, with $\overline{AB} \cong \overline{AC}$.

$\odot E$, $\overline{AD} \perp \overline{BC}$, $\overline{EF} \perp \overline{AC}$,

$AF = 6$, $ED = 1$

Find: **a** The radius of the circle

b The perimeter of $\triangle ABC$



16 Two chords intersect inside a circle. Prove that if a diameter drawn through the intersection point bisects the angle formed by the chords, then the chords are congruent. (Hint: Prove that the chords are equidistant from the center of the circle.)

10.3 ARCS OF A CIRCLE

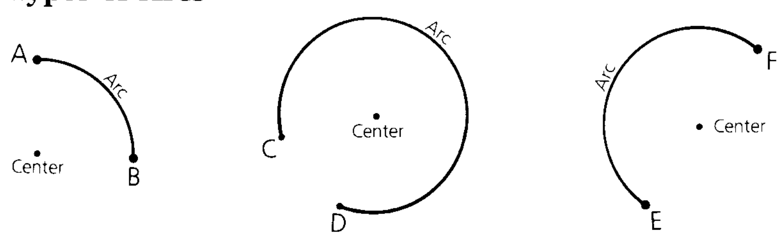
Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Part One: Introduction

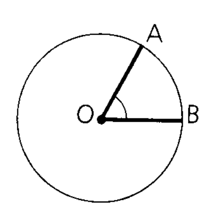
Types of Arcs



Definition An ***arc*** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

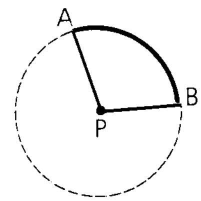
Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A ***central angle*** is an angle whose vertex is at the center of a circle.



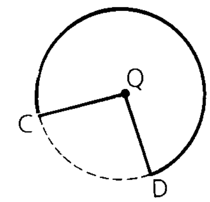
Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

Definition A ***minor arc*** is an arc whose points are on or between the sides of a central angle.



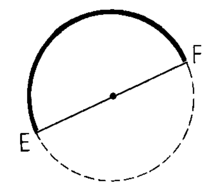
Central angle APB determines minor arc AB.

Definition A ***major arc*** is an arc whose points are on or outside of a central angle.



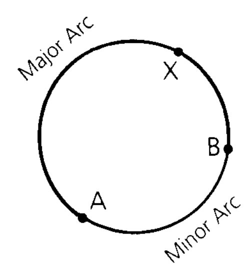
Central angle CQD determines major arc CD.

Definition A ***semicircle*** is an arc whose endpoints are the endpoints of a diameter.

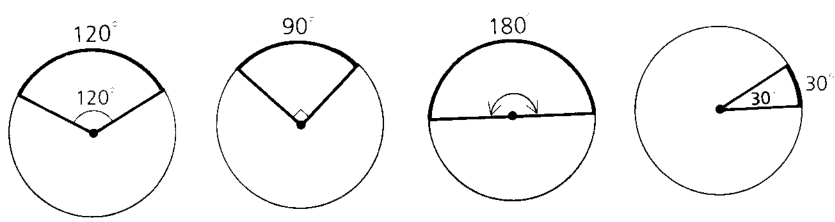


Arc EF is a semicircle.

The symbol $\frown$ is used to label arcs. The minor arc joining A and B is called $\widehat{AB}$. The major arc joining A and B is called $\widehat{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



The Measure of an Arc

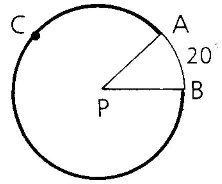


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

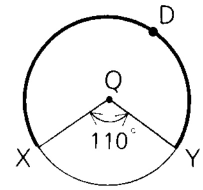
Example

a *Given:* $m\overset{\frown}{AB} = 20$
Find: $m\overset{\frown}{ACB}$



$$m\overset{\frown}{ACB} = 360 - 20$$
$$= 340$$

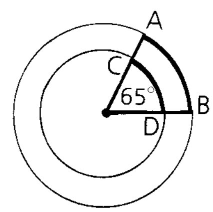
b *Given:* $m\angle XQY = 110$
Find: $m\overset{\frown}{XDY}$



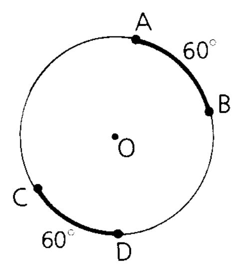
$m\overset{\frown}{XY} = m\angle XQY = 110$
Therefore, $m\overset{\frown}{XDY} = 360 - 110$
$= 250$

Congruent Arcs

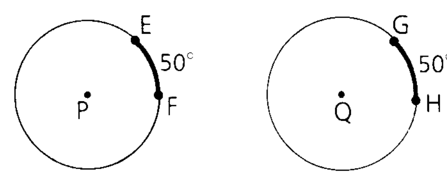
Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?



Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

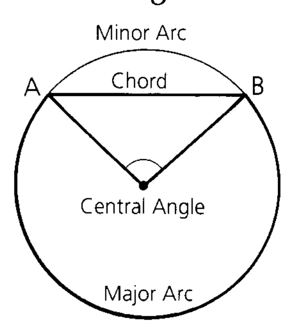


If $\odot P \cong \odot Q$, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

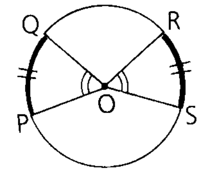
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.

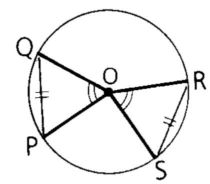


Theorem 79 ***If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.***



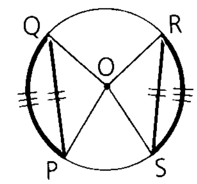
Theorem 80 ***If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.***

Theorem 81 ***If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.***



Theorem 82 ***If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.***

Theorem 83 ***If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.***

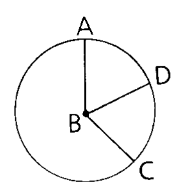


Theorem 84 ***If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.***

To summarize, in the same circle or in congruent circles, congruent chords $\Leftrightarrow$ congruent arcs $\Leftrightarrow$ congruent central angles.

Part Two: Sample Problems

Problem 1 Given: $\odot B$;
D is the midpt. of $\overset{\frown}{AC}$.
Conclusion: $\overrightarrow{BD}$ bisects $\angle ABC$.

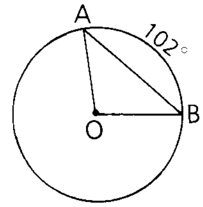


Proof

1 $\odot B$; D is the midpt. of $\overset{\frown}{AC}$.	1 Given
2 $\overset{\frown}{AD} \cong \overset{\frown}{DC}$	2 The midpoint of an arc divides the arc into two $\cong$ arcs.
3 $\angle ABD \cong \angle DBC$	3 If two arcs of a circle are $\cong$, then the corresponding central $\angle$s are $\cong$.
4 $\overrightarrow{BD}$ bisects $\angle ABC$.	4 If a ray divides an $\angle$ into two $\cong$ $\angle$s, then the ray bisects the $\angle$.

Problem 2 *If* m$\widehat{AB}$ = 102 *in* ⊙O, *find* m∠A *and* m∠B *in* △AOB.

Solution $\widehat{AB}$ = 102°, so ∠AOB = 102°.
The sum of the measures of the angles of a triangle is 180, so



$$\begin{aligned} m\angle AOB + m\angle A + m\angle B &= 180 \\ 102 + m\angle A + m\angle B &= 180 \\ m\angle A + m\angle B &= 78 \end{aligned}$$

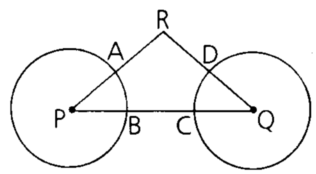
But $\overline{OA} \cong \overline{OB}$, so that ∠A ≅ ∠B.
Hence, m∠A = 39 and m∠B = 39.

Problem 3 **a** *What fractional part of a circle is an arc of 36°? Of 200°?*
b *Find the measure of an arc that is* $\frac{7}{12}$ *of its circle.*

Solution **a** 36° is $\frac{36}{360}$, or $\frac{1}{10}$, of a ⊙.
200° is $\frac{200}{360}$, or $\frac{5}{9}$, of a ⊙.

b There are 360° in a whole ⊙.
$\frac{7}{12}$ of 360 = $\frac{7}{12} \cdot \frac{360}{1}$ = 210

Problem 4 *Given:* ⊙s P *and* Q,
∠P ≅ ∠Q, $\overline{AR} \cong \overline{RD}$
Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that ⊙P ≅ ⊙Q.)



Proof

1 ⊙s P and Q	1 Given
2 ∠P ≅ ∠Q	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 If △, then △.
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtraction Property
6 ⊙P ≅ ⊙Q	6 ⊙s with ≅ radii are ≅.
7 $\widehat{AB} \cong \widehat{CD}$	7 If two central ∠s of ≅ ⊙s are ≅, then their intercepted arcs are ≅.

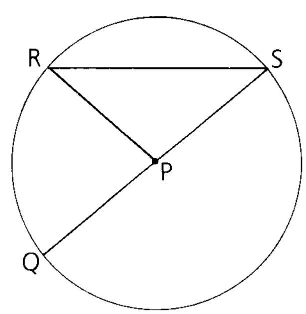
Part Three: Problem Sets

Problem Set A

1 Match each item in the left column with the correct term in the right column.

a $\widehat{QRS}$
b $\overline{QS}$
c $\widehat{RQS}$
d $\widehat{RS}$
e $\overline{RS}$
f ∠RPQ
g $\overline{PS}$

1 Radius
2 Diameter
3 Chord
4 Minor arc
5 Major arc
6 Semicircle
7 Central angle



2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

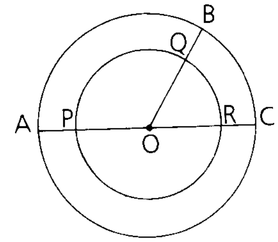
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

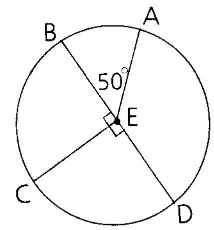
e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?



3 In circle E, find each of the following.

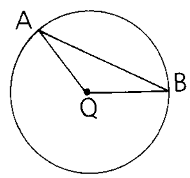
a $m\overset{\frown}{BC}$ **c** $m\overset{\frown}{ACD}$ **e** $m\overset{\frown}{ADC}$

b $m\overset{\frown}{AD}$ **d** $m\overset{\frown}{BAD}$



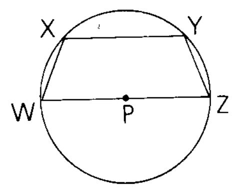
4 Given: $\odot Q$, $\angle A = 25°$

Find: $m\overset{\frown}{AB}$



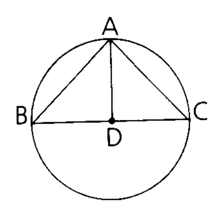
5 Given: $\odot P$, $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$

Conclusion: $\overline{WX} \cong \overline{YZ}$



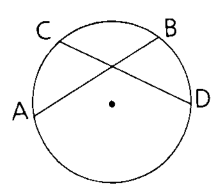
6 Given: $\odot D$, $\angle B \cong \angle C$

Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



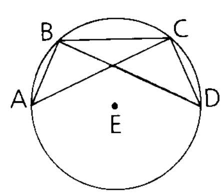
7 Given: $\overline{AB} \cong \overline{CD}$

Conclusion: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



8 Given: $\odot E$, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BD} \cong \overline{AC}$



Problem Set A, *continued*

9 What fractional part of a circle is an arc that measures

a 8 **c** 144

b 240 **d** 315

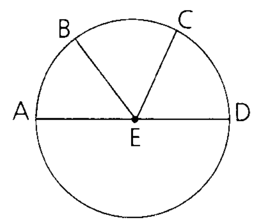
10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle **b** $\frac{5}{9}$ of its circle **c** 70% of its circle

Problem Set B

11 Given: $\overline{AD}$ is a diameter of ⊙E.
C is the midpoint of $\overset{\frown}{BD}$.
$m\overset{\frown}{AB} = 9x + 30$,
$m\overset{\frown}{CD} = 54 - x$

Find: $m\angle AEC$



12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

13 Find the length of each arc described. (The length is a fractional part of the circumference.)

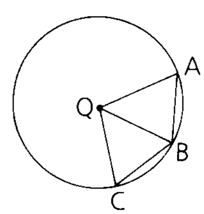
a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\overset{\frown}{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.

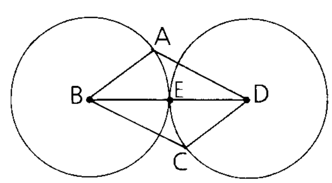
15 Given: ⊙Q;
B is the midpt. of $\overset{\frown}{AC}$.

Conclusion: $\angle A \cong \angle C$

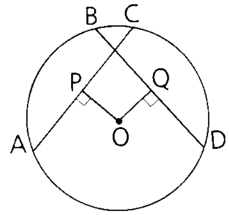


16 Given: ⊙B ≅ ⊙D,
$\overset{\frown}{AE} \cong \overset{\frown}{CE}$

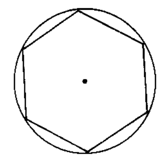
Prove: ABCD is a ▱.



17 Given: $\odot O$,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$



18 A polygon is *inscribed* in a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.



a A regular hexagon

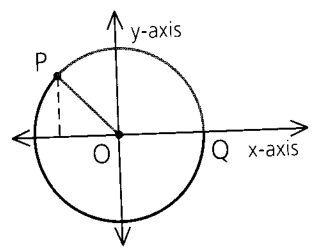
b A regular pentagon

c A regular octagon

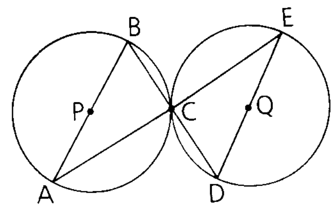
19 Point P is located at (−5, 5).

a Find the radius of $\odot O$.

b Find the measure of $\overset{\frown}{PQ}$.

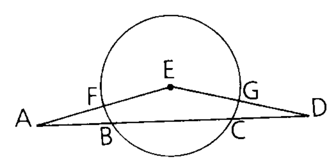


20 Given: $\odot P \cong \odot Q$,
$\overline{BC} \cong \overline{CD}$
Conclusion: $\angle A \cong \angle E$

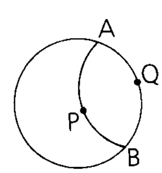


Problem Set C

21 Given: $\odot E$,
$\overline{AB} \cong \overline{CD}$
Conclusion: $\overset{\frown}{FB} \cong \overset{\frown}{CG}$

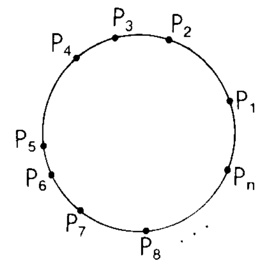


22 From point Q on circle P, an arc is drawn that contains point P. Find the measure of the arc AQB that is cut off.



Problem Set C, *continued*

23 If n points are selected on a given circle, find a formula

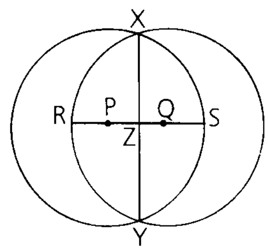


a For the number of chords that can be drawn between pairs of these points

b For the number of arcs formed—including major and minor arcs and semicircles (Hint: Draw circles and count arcs for $n = 1, 2, 3, \ldots$ until you see a number pattern.)

c For the measure of an arc formed by a side of a regular n-gon inscribed in the circle

24 Given: $\odot P \cong \odot Q$,
$XY = 8$,
$RP = QS = 1$
Find: PQ



25 Prove that if an equilateral polygon is inscribed in a circle, then it is equiangular.

26 Find, to the nearest tenth, the coordinates of point P on the circle with center O and radius 10, given that $m\widehat{PQ} = 40$. (Hint: Use trigonometry.)